

LEARNING MADE EASY



3rd Edition

Trigonometry

for
dummies[®]
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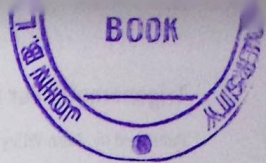
Understand the basics of
sines, cosines, and tangents

Learn how to use trigonometry
outside of the classroom

Graph functions and
solve tough formulas

Mary Jane Sterling

Master Pi Maker



Trigonometry

3rd Edition

by Mary Jane Sterling

for
dummies[®]
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